Subject: Probability and random processes		**Electrical Eng. Dept.**
Final-Term exam (A) - Time: 3 Hour		**Faculty of Engineering**
Semester: Sep. 2013 / Jan. 2014		**Benha University**

Answer the following questions:

1- True or false section

a.	The expectation of a random variable uniformly distributed over (a, b) is equal to (b+a).	(F)	1pt.
b.	If a and b are constants and ***X*** is a random variable and ***Y=aX+b***, then $f_Y(y)=\frac{1}{\lvert a\rvert}f_X\left(\frac{y-b}{a}\right)$.	(T)	1pt.
c.	If a and b are constants and X is a random variable , then $Var(aX+b)=a^2Var(X)$	(T)	1pt.
d.	The expected value of a product of two independent random variables is E(XY) = E(X)E(Y)	(T)	1pt.
e.	A continuous random variable is a random variable that can assume only countable values	(F)	1pt.
f.	If A is an event of a sample space with *P*(A)=*P*(A^c), then *P*(A)=0.5	(T)	1pt.
g.	If A and B are any two events of a sample space S, then the multiplication rule is: *P*(A and B)=*P*(A). *P*(B)	(F)	1pt.
h.	If two events are mutually exclusive, they are also independent	(F)	1pt.
i.	The mean of a continuous random variable X is found by multiplying X by its own probability density function and then integrate the product over all values of X; that is $\mu=\int_{-\infty}^{\infty}xf_X(x)dx$	(T)	1pt.
j.	Two events A and B are said to be independent if *P*(A and B)= *P*(A)+*P*(B)	(F)	1pt.

2- Choose the correct answer (put circle on the correct answer)

a.	If the continuous random variable *X* is uniformly distributed with a mean of 60 and a variance of 300. The probability that *X* lies between 50 and 80 is: **A** 1/4 **B** 1/3 **C** 1/2 **D** 2/3 **E** none of the above	2pt.
b.	If a random variable *X* has a probability density function $f(x)=\begin{cases}\frac{1}{16}(3x^2+4) & 0\le x\le 2\\ 0 & elsewhere\end{cases}$; then the variance of X is closest to : **A** 5/4 **B** 25/16 **C** 0.536 **D** 0.304 **E** none of the above	2pt.
c.	The continuous random variable *X* has probability density function *f* given by: $f(x)=\begin{cases}\left\lvert\frac{1}{4}(2-x)\right\rvert & 0\le x\le 4\\ 0 & \text{otherwise}\end{cases}$; *then* $E(X^2)$ is equal to : **A** $\frac{4}{3}$ **B** $\frac{16}{3}$ **C** 6 **D** 54 **E** none of the above	2pt.
d.	If random variable *X* has a mean $\mu_X=5$ and a standard deviation $\sigma_X=4$, and $Y=2-2X$ then: **A** $\mu_Y=8$ and $\sigma_Y=4$ **B** $\mu_Y=-8$ and $\sigma_Y=8$ **C** $\mu_Y=-8$ and $\sigma_Y=16$ **D** $\mu_Y=-8$ and $\sigma_Y=-8$	2pt.
e.	Let *X* be a real-valued, continuous random variable and let $Y=X^2$. Then, If $y\ge 0$, then the cumulative distribution function $F_y(y)$: **A** $F_x(-\sqrt{y})-F_x(\sqrt{y})$ **B** $F_x(\sqrt{y})-F_x(-\sqrt{y})$ **C** $F_x(\sqrt{y})+F_x(-\sqrt{y})$ **D** $-F_x(\sqrt{y})-F_x(-\sqrt{y})$	2pt.

3.	**a.**	If P(A)=0.6, P(B)=0.5 and P(A or B) =0.8. What is the P(A and B)?. P(A and B)= P(A) + P(B) - P(A or B) = 0.6 + 0.5 - 0.8 = 0.3	**2.5 pt.**
	b.	Consider the probability mass function $P(x) = \frac{6-\lvert x-7\rvert}{36}$ for x=2, 3, 4, 5,….,12. What will be $p(4 < x < 6)$?. $p(4 < x < 6) = p(x = 5) = P(5) = \frac{6-\lvert 5-7\rvert}{36} = \frac{4}{36}$	**2.5 pt.**
	c.	A telemarketer selling service contracts. He has sold 10 in his last 100 calls (***p* =.1**). If he calls **15** people tonight, what's the probability of: A. No sales? B. At least 2 sales? **Binomial distribution** $p(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$ n=15 p=0.1 A. $p(0) = \frac{15!}{(0!)(15!)} 0.1^0 (1-0.1)^{15-0} = 0.206$ B. $p(1) = \frac{15!}{1!\,(14)!} 0.1^1 (1-0.1)^{14} = 0.3432$ $p(\text{at least two sales}) = 1 - [p(0) + p(1)] = 1 - [0.206 + 0.3432] = 0.4508$	**2.5 pt.**
	d.	Data packets arrive at a network node on the average at a rate of 60 packets per minute. What is the probability of 5 packets arriving in 4 seconds? **Poisson Distribution** $p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ λ =60 packets per minute = 4 packet per 4 seconds $p(\text{5 packets arriving in 4 seconds}) = \frac{4^5 e^{-4}}{5!} = 0.1563$	**2.5 pt.**

4.		A continuous random variable X has CDF $F(x) = \begin{cases} a & x \le 0, \\ x^2 & 0 < x \le 1, \\ b & x > 1. \end{cases}$	
	a.	Determine the constants a and b. $a = 0, \quad where \lim_{x\to-\infty} F(x) = 0$ $b = 1, \quad where \lim_{x\to\infty} F(x) = 1$	**2 pt.**
	b.	Find the *pdf* of X. Be sure to give a formula for $f_X(x)$ that is valid for all x. $f_x(x) = \frac{d}{dx} F(x) = \begin{cases} 0 & x \le 0, \\ 2x & 0 < x \le 1 \\ 0 & x > 1. \end{cases}$	**2 pt.**
	c.	Calculate the expected value of X. $E[x] = \int_{-\infty}^{\infty} x f_x(x) dx = \int_0^1 2x^2 dx = \frac{2x^3}{3} \vert_0^1 = 2/3$	**2 pt.**
	d.	Calculate the standard deviation of X. $E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(x) dx = \int_0^1 2x^3 dx = \frac{2x^4}{4} \vert_0^1 = \frac{1}{2}$ $\sigma^2 = E[x^2] - E^2[x] = 1/18$	**2 pt.**

	e.	Find a form for $f_x(x\|X>0)$. $f_x(x/X>0)=\frac{f_x(x)}{p(X>0)}=f_x(x)=\begin{cases}0 & x\le 0,\\ 2x & 0<x\le 1\\ 0 & x>1.\end{cases}$	**2 pt.**

5.		In most communication systems, the noise (X) is treated as an additive Gaussian noise (random variable with normal distribution). If the noise mean is equal to zero and the variance is equal to 4. $z=\frac{x-\mu}{\sigma}=\frac{x}{2}$	
	a.	Find the probability $P(X>0.5)$. $p(x>0.5)=p(z>0.25)=1-p(z<0.25)=1-F_z(0.25)=0.4013$	**3 pt.**
	b.	Find the probability $P(X<-0.5)$. $p(x<-0.5)=p(z\le -0.25)=p(z>0.25)=1-F_z(0.25)=0.4013$	**3 pt.**
	c.	Find the probability $P(-2<X<2)$. $p(-2<x<2)=p(-1<z<1)=1-2F_z(-1)=-1+2F_z(1))=0.6826$	**3 pt.**
	d.	Use the Chebychev inequality to get an upper limit for $P(\|X-\mu\|\ge 2\sigma)$ Chebychev inequality states that $P(\|X-\mu\|\ge\varepsilon)\le\frac{\sigma^2}{\varepsilon^2}$, So $P(\|X-\mu\|\ge 2\sigma)\le\sigma^2/4\sigma^2=1/4$	**3 pt.**

6.	**a.**	Suppose the random variable (X) is uniformly distributed on the interval from 1 to 2. Compute the pdf and expected value of the random variable Y = 1-X . **In general** $Y=aX+b$ Suppose $a>0$. $F_Y(y)=P(Y(\xi)\le y)=P(aX(\xi)+b\le y)=P\left(X(\xi)\le\frac{y-b}{a}\right)=F_X\left(\frac{y-b}{a}\right)$. and $f_Y(y)=\frac{1}{a}f_X\left(\frac{y-b}{a}\right)$. On the other hand if $a<0$. then $F_Y(y)=P(Y(\xi)\le y)=P(aX(\xi)+b\le y)=P\left(X(\xi)>\frac{y-b}{a}\right)$ $=1-F_X\left(\frac{y-b}{a}\right)$, and hence $f_Y(y)=-\frac{1}{a}f_X\left(\frac{y-b}{a}\right)$. then, for all a $f_Y(y)=\frac{1}{\|a\|}f_X\left(\frac{y-b}{a}\right)$. ***Where X is uniformly distributed on the interval from 1 to 2. And a=-1 and b=1; then*** $f_y(y)=\begin{cases}1 & -1<y\le 0\\ 0 & elsewher\end{cases}$ uniform distribution on the interval from -1 to 0 $E[y]=E[1-x]=1-E[x] \quad =1-1.5=-0.5$	**4 pt.**

	b.	Use the moment generating function ($M(t) = E\left[e^{tX}\right]$) to get formulas for the mean, mean-square value and the variance of the binomial distribution. $M(t) = E\left[e^{tX}\right]$ $= \sum_{k=0}^{n} e^{tk} \binom{n}{k} p^k (1-p)^{n-k}$ $= \sum_{k=0}^{n} \binom{n}{k} (pe^t)^k (1-p)^{n-k}$ $= (pe^t + 1 - p)^n$ $M'(t) = n(pe^t + 1 - p)^{n-1} pe^t$ $M''(t) = n(n-1)(pe^t + 1 - p)^{n-2} (pe^t)^2 + n(pe^t + 1 - p)^{n-1} pe^t$ ■ Mean = $M'(0)$ = np ■ E[X^2] = $M''(0)$ = n(n-1)p^2 + np ■ Variance = E[X^2] – (E[X])^2 = n(n-1)p^2 + np –(np)^2 = np(1-p)	**4 pt.**